

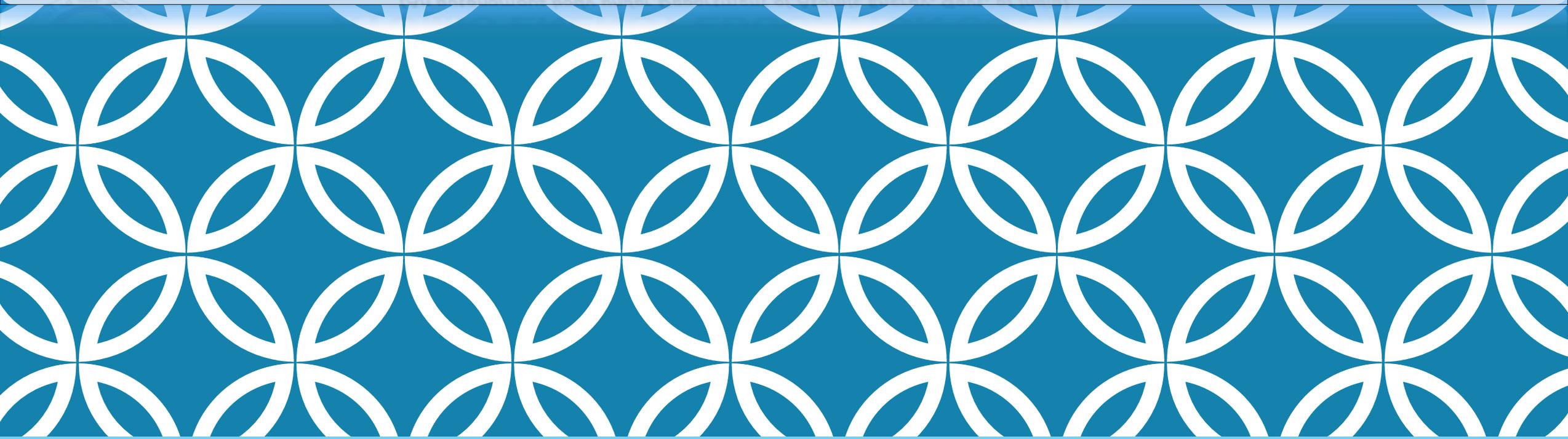


परमाणु ऊर्जा शिक्षण संस्था

(परमाणु ऊर्जा विभाग का स्वायत्त निकाय, भारत सरकार)

ATOMIC ENERGY EDUCATION SOCIETY

(An autonomous body under Department of Atomic Energy, Govt. of India)



CLASS: XI
SUBJECT: MATHEMATICS
CHAPTER: PERMUTATIONS AND COMBINATIONS
MODULE-3

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In this module ,we will learn

- Combinations
- Relation between permutations and combinations
- Applications of combinations
- Examples on permutations and combinations



Let's take an example,

A team of two players is to be formed from three players P,Q,R

In fact ,there are only three possible ways PQ,QR,PR as PQ,QP is the same selection QR,RQ and PR,RP are also the same



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- In the above example ,while finding the number of possibilities the order is not taken into consideration .
- Hence the number of selections that can be made from a group of given objects without reference to the order of objects in that group is known as combinations.



Combinations:

If r objects are to be chosen out of n objects where the order of arrangements is not important is given by the formula,

$${}^n C_r = \frac{n!}{r!(n-r)!}, 0 \leq r \leq n$$



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- The relation between the permutations and combinations is n_{p_r} and n_{c_r} :

We know that is $n_{p_r} = \frac{n!}{(n-r)!}$ and $n_{c_r} = \frac{n!}{r!(n-r)!}$,

Hence $n_{p_r} = r! n_{c_r}$, $0 < r \leq n$.

- The properties of n_{c_r} :

(i) $n_{c_r} = n_{c_{n-r}}$

(ii) $n_{c_n} = n_{c_0} = 1$

(iii) $n_{c_x} = n_{c_y}$, implies $x = y$ or $x + y = n$



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$$(iv) n_{c_r} + n_{c_{r-1}} = n + 1_{c_r}$$

Examples:

1. If $n_{c_9} = n_{c_8}$, find $n_{c_{17}}$

by the property, $n_{c_x} = n_{c_y}$

on comparing with this property, $x = 9$ and $y = 8$

$$n = x + y = 8 + 9 = 17$$

$$\text{Hence } n_{c_{17}} = 17_{c_{17}} = 1$$



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2. Find ${}^5C_3 + {}^5C_2$

The given question is of the form of

$$n_{C_r} + n_{C_{r-1}}$$

comparing with it $n = 5, r = 3$

Hence by applying with

$$n_{C_r} + n_{C_{r-1}} = n + 1_{C_r}, \text{ the ans is } {}^6C_3 = 20$$



3. How many chords can be drawn through the 21 points of a circle?

Ans:

To draw a chord two points are required, hence number of

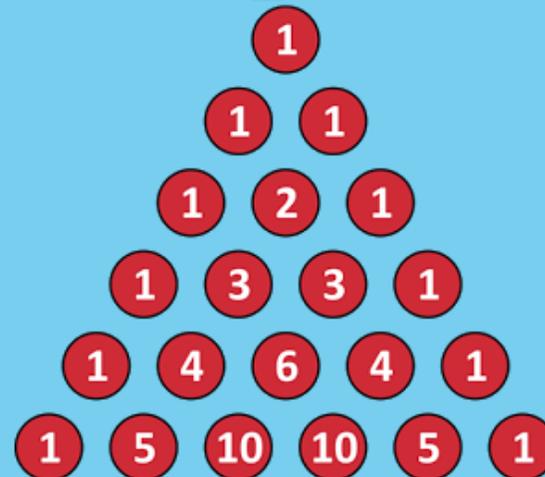
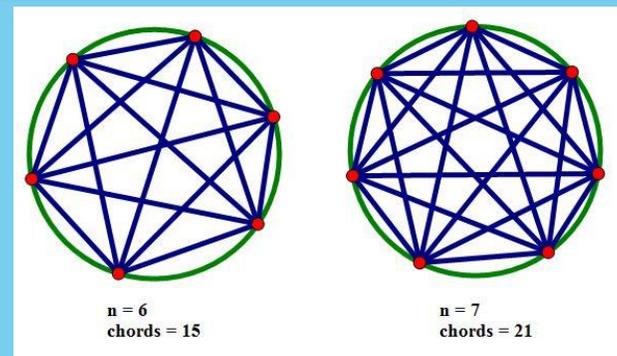
chords which can be drawn is ${}_{21}C_2 = \frac{21 \cdot 20}{2} = 210$

Application of n_{C_r} :

Useful for finding the coefficients

Of binomials, shown here as

Pascal's triangle .





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4. If $n_{p_r} = 840$ and $n_{c_r} = 35$, then r is ?

We know that $n_{p_r} = r! n_{c_r}$,

Hence $840 = r! \times 35$, therefore

$$r! = \frac{840}{35} = 24, \text{ Hence } r = 4$$

5. Find the number of triangles that are formed by choosing the vertices from a set of 12 points, seven of which lie on the same line is ?

Ans: The number of triangles which can be formed from 12 points is ${}^{12}C_3$.

Out of these 7 points lie on the same line, hence the triangles formed from these points are to be excluded, 7C_3

$$\text{Hence the number of triangles is } {}^{12}C_3 - {}^7C_3 = 220 - 35 = 185$$



6. Find, the number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines.

Ans: To form a parallelogram we require a pair of line from a set of 4 lines and another set of 3 lines .Hence required number of parallelograms
 $= 4_{C_2} \times 3_{C_2} = 18$

7. A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected?

Ans: Out of 5 black ,2 black balls can be selected in 5_{C_2} ways and out of 6 red balls , 3 red balls can be selected in 6_{C_3} , hence total number of ways in which 2 black and 3 red balls can be selected is $5_{C_2} \times 6_{C_3} = 200$



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8. A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of (i) Exactly 3 girls (ii) at least 3 girls (iii) at most 3 girls?

Ans: (i) If exactly 3 girls are to be selected from 4 girls, it can be done in 4C_3 ways, remaining 4 should be boys and these can be done in 9C_4 boys. Hence required number of ways = ${}^4C_3 \times {}^9C_4 = 504$



(ii) We have to select at least 3 girls.

So the committee consists of 3 girls and 4 boys or 4 girls

and 3 boys then ,number of ways of selection = ${}^4C_3 \times {}^9C_4 + {}^4C_4 \cdot {}^9C_3$

(iii) We have to select atmost 3 girls.so the committee consisits of no girl and 7 boys or 1 girl and 6 boys or 2 girls and 5 boys or 3 girls and 4 boys

Hence number of selections = ${}^4C_0 \times {}^9C_7 + {}^4C_1 \times {}^9C_6 + {}^4C_2 \times {}^9C_5 + {}^4C_3 \times {}^9C_4$



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9. How many words, with or without meaning, each of 2 vowels and 3 consonants can be formed from the letters of the word 'DAUGHTER' ?

Ans: There are 8 letters in the word DAUGHTER including 3 vowels and 5 consonants. We have to select 2 vowels out of 3 vowels and 3 consonants out of 5 consonants.

Hence number of selections = ${}^3C_2 \times {}^5C_3 = 3 \times 10 = 30$

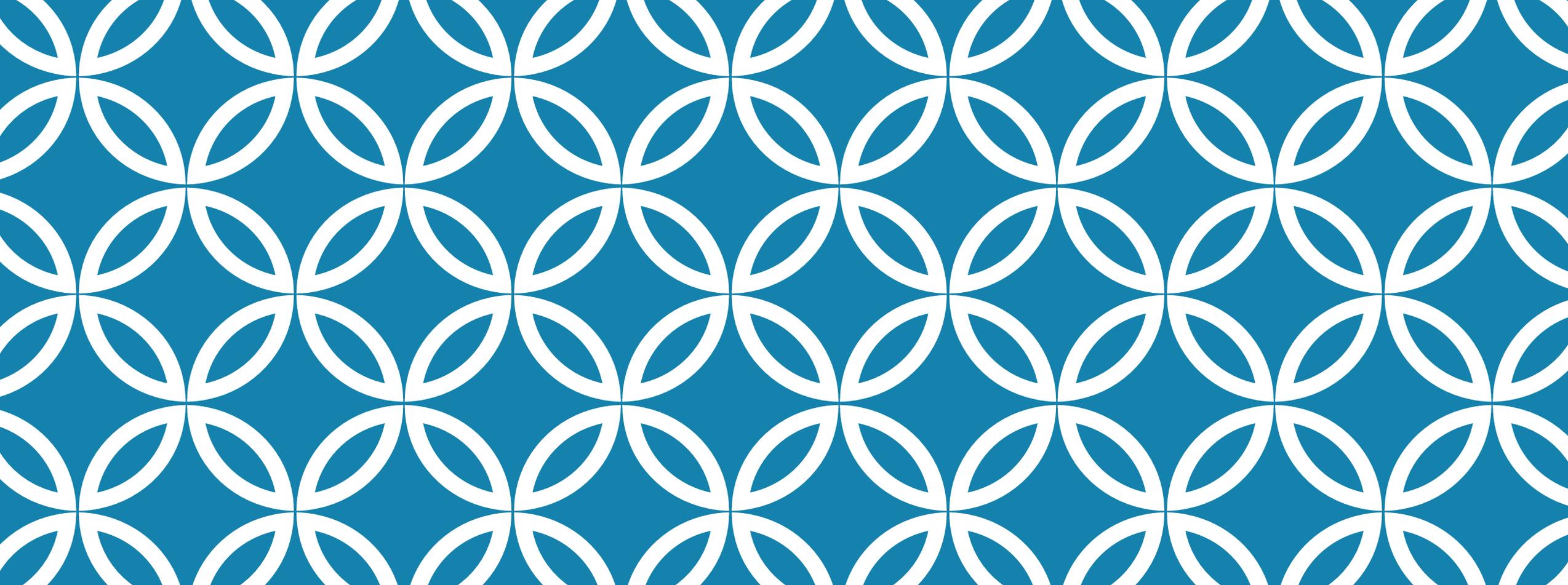
Now each word contains 5 letters which can be arranged among themselves in 5! Ways.

So, total number of words = $5! \times 30 = 120 \times 30 = 3600$.



PRACTICE QUESTIONS

1. How many words with or without meaning can be formed from the letters of the word 'TRIANGLE' so that no vowels are together.
2. A box contains 5 red balls and 6 black balls. In how many ways can 6 balls be selected so that there are at least two balls of each colour?
3. If a convex polygon has 44 diagonals, then find the number of its sides.
4. Find the $15C_8 + 15C_9 - 15C_6 - 15C_7$
5. There are 10 lamps in a hall. Each one of them can be switched on independently. Find the number of ways in which the hall can be illuminated.



**THANK YOU
HAPPY LEARNING**

REFERENCES:

**NCERT TEXT BOOK,
NCERT EXEMPLAR,
DIKSHA WEBSITE**